

# FINITE ELEMENT ANALYSIS OF PRESTRESSED CONCRETE BEAMS

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**Abstract:** Concrete structural components exist in buildings and bridges in different forms. Understanding the response of these components during loading is crucial to the development of an overall efficient and safe structure. Different methods have been utilized to study the response of structural components. Experimental based testing has been widely used as a means to analyse individual elements and the effects of concrete strength under loading. While this is a method that produces real life response, it is extremely time consuming, and the use of materials can be quite costly. The use of finite element analysis to study these components has also been used. Unfortunately, early attempts to accomplish this were also very time consuming and infeasible using existing software and hardware. In recent years, however, the use of finite element analysis has increased due to progressing knowledge and capabilities of computer software and hardware. It has now become the choice method to analyze concrete structural components. The use of computer software to model these elements is much faster, and extremely cost-effective. To fully understand the capabilities of finite element computer software, one must look back to experimental data and simple analysis. Data obtained from a finite element analysis package is not useful unless the necessary steps are taken to understand what is happening within the model that is created using the software. Also, executing the necessary checks along the way, is key to make sure that what is being output by the computer software is valid. By understanding the use of finite element packages, more efficient and better analyses can be made to fully understand the response of individual structural components and their contribution to a structure as a whole. This paper is a study of prestressed concrete beams using finite element analysis to understand the response of prestressed concrete beams due to transverse loading.

**Keywords:** - Finite Element Analysis, Prestressed Concrete Beam, Software Indulge (ANSYS 12.1)

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## SCOPE OF STUDY

The scope of this study is limited to the determination of the structural static properties such as deflections and stress distributions. For that Rectangular prestressed concrete beam is taken for the analysis. The ANSYS 12.1 package program is used as a tool of this finite element analysis. The prestressed concrete beam is modeled as simply supported. Isotropic materials are used over the beam sections.

## 1. INTRODUCTION

### 1.1 FINITE ELEMENT ANALYSIS

Finite element analysis is an effective method of determining the static performance of structures for three reasons which are saving in design time, cost effective in construction and increase the safety of the structure. Previously, it is necessary to use advanced mathematical methods in analysis large structures, such as bridges, tall buildings and other. More accuracy generally required more elaborate techniques and therefore a large fraction of the designer's time could be devoted to mathematical analysis. Finite element methods free designer's from the need to concentrate on mathematical calculation and allow them to spend more time on accurate representation of the intended structure and review of the calculated performance (Smith, 1988). Furthermore, by using the programs with interactive

graphical facilities, it is possible to generate finite element models of complex structures with considerable ease and to obtain the results in a convenient, readily assimilated form. This may save valuable design time. More accurate analysis of structure is possible by the finite element method leading to economics in materials and construction also in enhancing the overall safety (De Salvo and Swanson, 1985).

However, in order to use computer time and design time effectively, it is important to plan the analysis strategy carefully. Before a series of dynamic tests carry out in the field, a complete three-dimensional finite element models are developed for each bridges, prior to its testing. The results from these dynamic analyses are used to select instrument positions on the bridge and predict static displacement. Then, they are calibrated using the experimental frequencies and mode shapes. The frequencies and mode shapes mainly are used to provide a basis for the study of the influence of certain parameters on the dynamic response of the structure, the influence of secondary structural elements, the cracking of the deck slabs, the effects of long-term concrete creep and shrinkage and so on (Paultre and Proulx, 1995). Besides, more sophisticated methods based on finite element or finite strip representation have been used by some researchers to study the dynamic behavior of bridges, Fam (1973) and Tabba (1972) studied the

behavior of curved box section bridges using the finite element method for applied static and dynamic loads. A three-dimensional finite element analysis program was developed for curved cellular structures. Solutions of several problems involving static and dynamic responses were presented using the proposed and other sophisticated methods of analysis. An experimental study conducted on two curved box girder Plexiglas models confirmed the reliability of the proposed methods of analysis.

## 1.2 PRESTRESSING BASIC CONCEPT OF PRESTRESSING

Prestress concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that stresses resulting from external loads are counteracted to a desired degree. In reinforced concrete members, the prestress is commonly introduced by tensioning the steel reinforcement.

## ADVANTAGES OF PRESTRESSED CONCRETE

Prestressed concrete offers great technical advantages in comparison with other forms of construction, such as reinforced concrete and steel. In the case of fully prestressed members, which are free from tensile stresses under working loads, the cross section is more effectively utilized when compared with a reinforced concrete section which is cracked under working loads. Within certain limits, a permanent dead-load may be counteracted by increasing the eccentricity of the prestressing force in a prestressed structural element, thus effecting savings in the use of materials. Prestressed concrete members possess improved resistance to shearing forces, due to the effect of compressive prestress, which reduces the principal tensile stress. The use of curved cables, particularly in long-span members, helps to reduce the shear forces developed at the support sections.

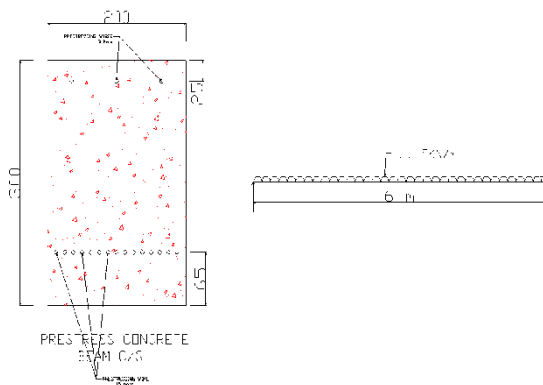
A prestress concrete flexural member is stiffer under working loads than a reinforced concrete member of the same depth. However, after the onset of cracking, the flexural behavior of a prestressed member is similar to that of a reinforced concrete member. The use of the high strength concrete and steel in prestress members results in lighter and slender members than is possible with reinforced concrete. The two structural features of prestressed concrete namely high strength concrete and freedom from cracks, contributes to the improved durability of the structures under aggressive environmental conditions. Prestressing of concrete improves the durability of material for energy absorption under impact loads. The ability to resist repeated working loads has been proved to be as good in prestressed as in reinforced concrete.

The economy of the prestressed concrete is well established for long span structures. According to Dean<sup>6</sup> standardized precast bridge between 10 to 30 m long precast prestressed piles have proved to be more economical than steel and reinforced concrete in the United States. According to Abeles<sup>7</sup>, pre cast prestress concrete is economical for floors, roofs and bridges of spans up to 30 m, and for cast in situ work, up to 100 m. In the long-span range, prestressed concrete is generally more economical than reinforced concrete and steel.

## 1.3 RECTANGULAR PRESTRESSED CONCRETE BEAM

### Problem Analysis

A rectangular beam of cross-section 300 mm deep and 200 mm wide is Prestressed by means of 15 wires of 5 mm diameter located 65 mm from bottom of the beam and 3 wires of diameter of 5 mm, 25 mm from top. Assuming the prestress in the steel as 840 N/mm<sup>2</sup>. If a uniformly distributed live load of 6 KN/m is imposed, evaluate the maximum working stress in concrete. The density of concrete is 24 KN/m<sup>3</sup>.



Rectangular Prestressed Concrete Beam  
DETAILS OF PRESTRESSED CONCRETE BEAM GIRDER

### MATERIAL PROPERTIES

- \* Grade of concrete = M-40
- \* Cube strength of concrete  $f_{cu} = 40 \text{ N/mm}^2$
- \* Modulus Elasticity of Concrete  $E_c = 31622.77 \text{ N/mm}^2$
- \* 5 mm dia. High tensile wires having characteristic strength  $f_{pu} = 1400 \text{ N/mm}^2$
- \* Modulus Elasticity of high tensile wires  $E_c = 210 \text{ kN/mm}^2$

### LOADING CONSIDERED

- Live Load = 5 kN/m
- Dead load = 1.44 kN/m

Eccentricity  $e = (150-100) = 50$

Prestressing force  $P = (840 \times 18 \times 19.7) = 3 \times 10^5 \text{ N}$

Area of Cross section  $A = (300 \times 200) = 6 \times 10^4$  mm<sup>2</sup>

Moment of inertia  $I = (200 \times 300^3)/12 = 45 \times 10^7$  mm<sup>4</sup>

Self-Weight moment  $M_g = (1.44 \times 6^2)/8 = 6.48$  KNm

Live-Load moment  $M_q = (6 \times 6^2)/8 = 27$  KNm

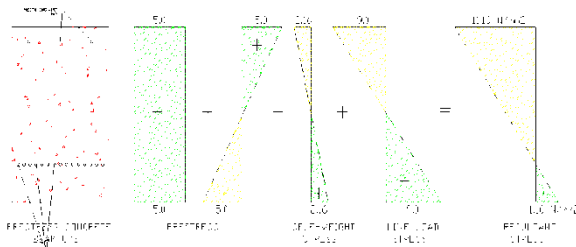
Direct stress due to prestress  $(P/A) = (3 \times 10^5 / 6 \times 10^4) = 5$  N/mm<sup>2</sup>

Bending stress due to prestress  $(Pe/Z) = \{3 \times 10^5 \times 50 / 3 \times 10^6\} = 5$  N/mm<sup>2</sup>

Self-Weight stress  $M_g / Z = (6.48 \times 10^6 / 3 \times 10^6) = 2.16$  N/mm<sup>2</sup>

Live load stress  $M_q / Z = (27 \times 10^6 / 3 \times 10^6) = 9$  N/mm<sup>2</sup>

The Resultant stresses due to (self-weight + prestress + live load) are shown in fig. Maximum working stress in concrete = 11.16 N/mm<sup>2</sup> (compression).



STRESS DISTRIBUTION AT MID-SPAN  
Prestressed Section

Span : 6000mm

Downward Deflection

$$D_{max} = 5wl^4/384EI$$

$$= (5 \times 7.44 \times 6000^4) / (384 \times 12162.606 \times 450 \times 10^6)$$

$$= 22.939 \text{ mm}$$

Upward Deflection Due to Prestress

$$P_{MAX} = -PeL^2/8EI$$

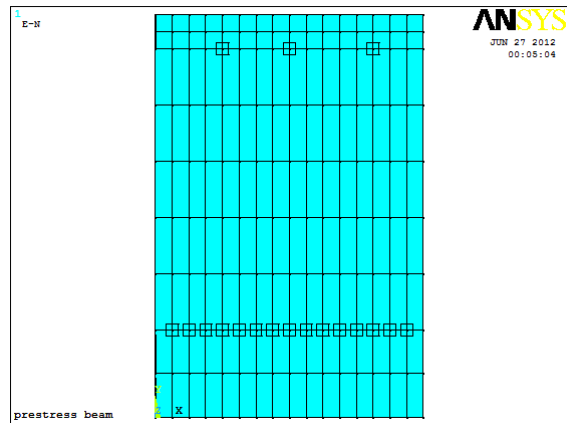
LENGTH OF GIRDER(mm)	PRESTRESSED SECTION	
	Calculation	ANSYS
6000 mm	10.6062 mm	10.457 mm

$$= -3 \times 10^5 \times 50 \times 6000^2 / 8 \times 12162.606 \times 450 \times 10^6$$

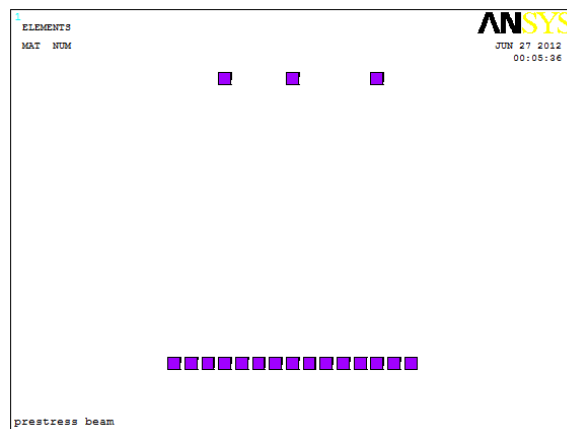
$$= -12.3328 \text{ mm}$$

$$\text{Net Downward Deflection} = 22.939 - 12.3328 = 10.6062 \text{ mm}$$

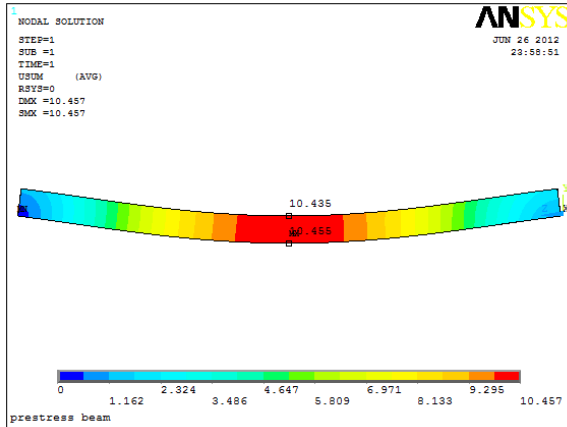
ANSYS 12.1 SOFTWARE OUTPUT



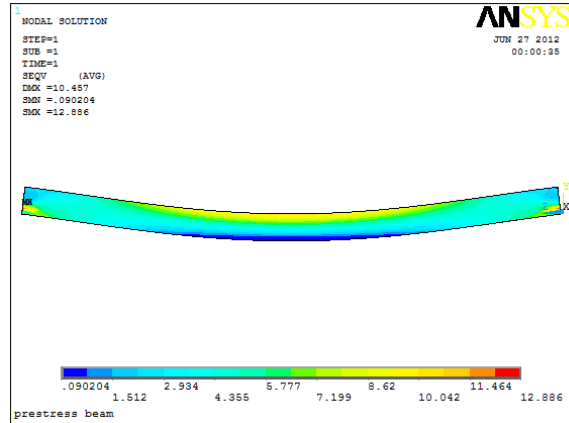
Cross Sectional Details



Section Showing the Position of Prestressing Wires



Deformations In Y Direction



Von Mises Stress Distribution In Z Direction

2. RESULT

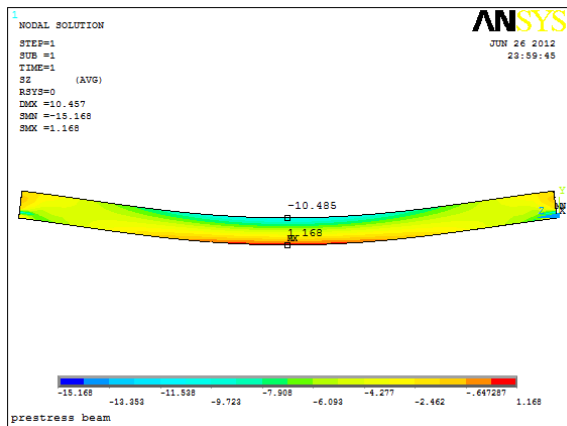
Deformation Table  
Stress Distribution Table

METHOD OF ANALYSIS	STRESSES N/mm <sup>2</sup>	
	TOP FIBER	BOTTOM FIBER
CALCULATION	-11.16	+1.16
ANSYS	-10.48	+1.16

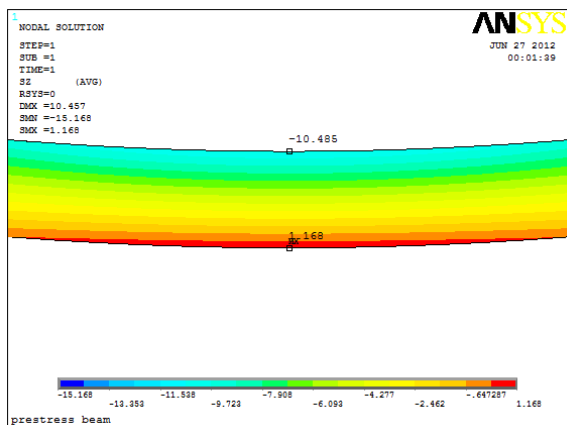
3. CONCLUSION

The following conclusions can be stated based on the evaluation of the analyses of the calibration model and the prestressed concrete beam.

- (1) The failure mechanism of a prestressed concrete beam is modelled quite well using FEA, and the failure load predicted is very close to the failure load calculated.
- (2) Deflections and stresses at the zero deflection point and decompression are modeled well using a finite element package.
- (3) Deflections and stresses at the centreline along with initial and progressive cracking of the finite element model compare well to theoretical results.
- (4) The load applied to cause initial cracking of the prestressed concrete beam compares well with hand calculations.
- (5) Flexural failure of the prestressed concrete beam is modeled well using a finite element package, and the load applied at failure is very close to hand calculated results.



Stress Distribution In Z Direction (Static Analysis)



Stress Distribution In Z Direction (Close View)

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